Mass Energy Equivalence

Due to the fact that mass changes as particles approach the speed of light, the Newtonian definition for momentum is only valid for classical mechanics. In order to account for special relativity, the relativistic mass of the particle must be taken into account, changing $p = mv$ into

$$p = \frac{m_o v}{\sqrt{1 - \frac{v^2}{c^2}}} \qquad (4)$$

This equation is used when the law of conservation of momentum is applied to relativistic situations.

Since momentum has been modified for relativistic speeds, it is only natural to modify the equation for energy to account for the dilation of mass at high speeds. Therefore, $E = mc^2$ becomes

$$E = \frac{m_o c^2}{\sqrt{1 - \frac{v^2}{c^2}}}$$

If a binomial expansion were to be performed

$$\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} = 1 + \frac{v^2}{2c^2} + \frac{3v^4}{8c^4} + \ldots$$

The above equation becomes

$$E = m_o c^2 \left(1 + \frac{v^2}{2c^2} + \frac{3v^4}{8c^4} + \ldots \right)$$

$$E = m_o c^2 + \frac{1}{2} m_o v^2 + \ldots$$

But relativistically, $E_k = ½m_o v^2$ therefore

$$E = m_o c^2 + E_k \qquad (5)$$

The rest energy, given by $E_o = m_o c^2$, is the energy that makes up the internal structure of that object.

Examples:

1. The beam of electrons in a television reaches speeds of approximately 9.2×10^7 m/s. Determine the momentum of the beam of electrons:
   - **a.** using classical mechanics
   - **b**. using relativity

2. In subatomic physics, it is convenient to use the electron volt (eV), rather than the joule, as the unit of energy. An electron moves at 0.860c in a laboratory. Determine the electron's rest energy, total energy, and kinetic energy in the laboratory frame, in electron volts.

Pg 583 #1-6 (even) Pg 584 #1-9 (odd) **11.3**